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The use of the Brinkman number for single phase forced convective heat transfer in microchannels

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Abstract—From a survey on studies on convective heat transfer in microchannels, the Brinkman number is proposed as a parameter for correlating the convective heat transfer parameters in microchannels. The proposal emerges from a dimensional analysis of the variables influencing the laminar forced convection in microchannels and it can explain the unusual behaviour of convective heat transfer in the laminar regime in microchannels. Its incorporation is also supported by an energy balance across the microchannel. The physical significance of the Brinkman number as applicable to microchannels and the role played in convective heat transfer in microchannels are elaborated. The limited experimental data reported in the literature for the laminar regime heat transfer seem to correlate with the number. A dimensionless geometric parameter is also proposed.

1. INTRODUCTION AND SURVEY

The increasing scales of circuit integration of electronic components accompanied by reducing feature size of IC chips have tremendously increased the problems associated with the dissipation of the generated heat. Hence the development of efficient cooling techniques for integrated circuit (IC) chips is one of the important contemporary applications of microscale heat transfer. Tuckermann and Pease [1] in 1982 demonstrated that IC chips can be effectively cooled by the forced convective flow of water through microchannels fabricated either directly on the IC chip (as illustrated in [2]) or in the circuit board. They noted that the convective heat transfer coefficient, h, for laminar flow through microchannels might be higher than for turbulent flow through conventionally-sized channels. Since then there has been an unprecedented upsurge of research in convective heat transfer through microchannels.

Choi *et al.* [3] have measured the friction factor, inner wall roughness, and h for laminar and turbulent flow of nitrogen gas in circular microtubes with inside diameters ranging from 3–81 μm. Their results indicated significant departures from the thermofluid correlations used for conventionally-sized tubes; the laminar fully-developed h values exhibited an Re dependence, while turbulent h values were larger than those predicted by the conventional correlations for smooth tubes. The Colburn and Petukhov analogies between momentum and energy transport were also not supported by their experimental data for microtubes.

Tuckermann and Pease [4] have presented a theory for optimising the design of microchannel water-cooled integral heat sinks for ICs. Their optimised configuration which was employed in their experimental investigations had a channel width of approximately 50 μm and a height of approximately 300 μm. Makihara *et al.* [5] have experimentally investigated the flow of liquids in 4.5–50.5 μm microcapillary tubes, and found that the measured values of the flow parameters agree with those calculated from the Navier–Stokes equations, indicating that the continuum assumptions did not break down.

Peng and Wang [6] have experimentally investigated the heat transfer characteristics of subcooled liquid flowing through microchannels with a rectangular cross-section of 0.6×0.7 mm. Their results showed that h was enhanced, and that the velocity and subcooling may influence the single-phase convection. In an investigation on water flow through channels with hydraulic diameter D_h in the range 0.133–0.367 mm and aspect ratio (H/W) in the range 0.333–1, Peng *et al.* [7] found that the upper bound of the laminar flow regime occurred in the Re range 200–700, and fully turbulent flow was reached in the range 400–1500. The transition Re and range were found to reduce with the reduction of the critical microchannel dimension, δ. (For a rectangular microchannel the critical dimension is the smaller of H and W.) Hence it was concluded that the geometric parameters significantly affect the flow and hence the convective heat transfer. In another paper [8], they reported that in the laminar regime, Nu is proportional to $Re^{0.62}$, while

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NOMENCLATURE

A', A	empirical constants in equations (8), (15) [-]
Br	Brinkman number [-]
c	specific heat of fluid [J kg^{-1} K^{-1}]
d	exponent of Br [-]
D	diameter of circular microchannel [m]
D_h	hydraulic diameter of microchannel [m]
Ec	Eckert number [-]
f_g	microchannel geometric function [-]
h	convective heat transfer coefficient [W m^{-2} K^{-1}]
h'	hypothetical convective heat transfer coefficient without the effect of Br [W m^{-2} K^{-1}]
H	height of microchannel [m]
k	thermal conductivity of fluid [W m^{-1} K^{-1}]
L	fundamental dimension for length
$\dot{m}_f$	fluid mass flow rate through microchannel [kg s^{-1}]
M	fundamental dimension for mass
Nu	Nusselt number [-]
Pr	Prandtl number [-]
Re	Reynolds number [-]
T	fundamental dimension for time
T_f	bulk mean fluid temperature [°C]
$T_{f_{in}}$	inlet temperature of fluid [°C]
T_w	local wall temperature [°C]
V_{av}	average velocity of fluid at particular axial location [m s^{-1}]
V_{max}	maximum velocity of fluid (at axis of microchannel) [m s^{-1}]
W	width of microchannel [m]
W_c	centre-to-centre distance between two consecutive microchannels [m].

Greek symbols

δ	critical microchannel dimension [m]
ΔT	wall–fluid temperature difference at a particular axial location [K]
ΔT_f	change in fluid temperature along flow [K]
Θ	fundamental dimension for temperature
μ	dynamic viscosity of fluid [N s m^{-2}]
ν	kinematic viscosity of fluid [m^2 s^{-1}]
Π	dimensionless number [-]
ρ	density of fluid [kg m^{-3}]
σ	square root of variance.

in the turbulent regime, the typical relation between Nu and Re is obtained, but with a different empirical constant.

Wang and Peng [9] explored the influence of liquid flow and thermal parameters, geometrical size, and structure on the convective flow of methanol and deionized water through rectangular microchannels, and found the variation of h with the wall temperature T_w, and that of Nu with Re for different geometries, liquid inlet temperatures, and velocities. The behaviour of h was found to be highly associated with the liquid flow or the heat transfer mode, suggesting the existence of separate heat transfer mechanism(s) accordingly, which are unidentified. The laminar and transition heat transfer were found to be unusual compared to the conventionally-sized situations. For $Re < 700$, Nu appeared to be a function of some other unknown variations besides Re. The reduction of Nu with increase in Re in the laminar zone and the approximately constant Nu in the transition zone was not explained. The heat transfer characteristics of both laminar and transition flow were found to be affected by the liquid temperature, velocity, and microchannel size. The heat transfer in the fully turbulent flow regime was found to be predicted by the Dittus–Boelter correlation by modifying the empirical constant coefficient from 0.023–0.00805.

Yu *et al.* [10] experimentally investigated the flow of dry nitrogen gas and water in microtubes with diameters 19, 52, and 102 μm for Re ranging from 250 to over 20 000 and Pr ranging from 0.7–5.0, and they found a reduction in the friction factor in the turbulent regime and that the heat transfer was enhanced. The Reynolds analogy was found not applicable in channels whose dimensions are of the order of the turbulent length scale, though the fluid can still be treated as a continuum. Their theoretical scaling analysis indicated the turbulent momentum and energy transport in the radial direction to be significant in the near-wall zone in a microtube. They developed an analogy by considering the turbulent eddy interacting with the walls as a frequent event thereby causing a direct mass and thermal energy transfer process between the turbulent lumps and the wall, similar to the nucleate boiling process or the eddies bursting phenomenon, which significantly alters the laminar sublayer region in turbulent flows through microtubes. Since even a small eddy diffusivity in the laminar sublayer region can contribute significantly to the heat transfer rate while having a negligible effect on momentum transfer, an eddy can carry heat a greater distance than momentum. This explanation accounts for the increased h and lower friction factors in turbulent flows through microtubes.

Peng and Peterson [11] have presented the variations of Nu with Re, and of h with T_w for various channel geometries, liquid inlet temperatures, and velocities for water flow. The relation between *Nu* and *Re* was found to be complicated in the laminar and transition regimes. In these regimes, *Nu* may decrease with an increase in *Re*, and *Nu* is apparently associated with other factors like the liquid temperature, velocity, and microchannel size. The variation of h with T_w was attributed to the small size of the channels. Though no way to properly compare the results of one microchannel size with another was reported, the heat transfer performance apparently improved as the microchannel size decreased. For $Re > 1000$, all the experimental data was found to fit along a fairly narrow region that traces a straight line on a log–log scale in which *Nu* increases with *Re*, which is the turbulent regime. Peng *et al.* [12] also reported experimental results for methanol flow and found the existence of a heat transfer or liquid flow mode transition region which is a function of the heating rate or T_w, as reported in [9].

Recently, Peng and Peterson [13] have compared their correlations between *Nu*, *Re*, *Pr*, and a dimensionless geometric parameter of the microchannels with their experimental data, for the laminar and turbulent regimes. The reduction of *Nu* with increase in *Re* in the laminar regime (as reported in [9, 11]) and the constant *Nu* in the transition regime (as reported in [9]) cannot be explained by their correlation. Hence their laminar regime correlation may be incomplete and an additional dimensionless number may have to be included to account for some other mechanism(s) which causes *Nu* to recede or remain constant with *Re*. The geometry of the microchannel plate and individual microchannels were found to have a significant effect on the single-phase convective heat transfer and flow characteristics (as reported in [7–9, 11]), and its effect on the laminar and turbulent convection was found to be different. The laminar heat transfer was found to depend on the aspect ratio (H/W) and the ratio of the hydraulic diameter to the centre-to-centre distance of the microchannels (D_h/W_c). The transition heat transfer was reported to be complicated.

Choquette *et al.* [14] have developed a computer code to evaluate the performance capabilities, power requirements, efficiencies, and for optimisation of the microchannel heat sink. Their results showed that significant reductions in the total thermal resistance are not achieved by designing for turbulent flow mainly because the significantly higher pumping power requirements realised offset the slight improvement in the overall thermal performance. This suggests the importance of proper design in microchannels for laminar flow conditions. In another recent theoretical study by Mala *et al.* [15], the possible importance of the electric double layer (EDL) effect on microchannels was proposed. They reported that the EDL thickness ranges from a few nanometers to several hundreds of nanometers, and calculated the effect on a microchannel separation distance of 25 μm. As this is an order of magnitude smaller than the channels used in experiments reported in the literature, the true influence of EDL on the convective heat transfer is uncertain.

In summary, it may be stated that the heat transfer behaviour in microchannels significantly differs from that in the conventionally-sized channels. Heat transfer is enhanced since the fluid conduction resistance is substantially reduced and the contact area between the IC chip and coolant is increased due to the small size of the channels. Though the liquid flow is not in the non-continuum regime for the microchannel dimensions under consideration, the behaviour of h is highly associated with the flow or heat transfer mode, suggesting a change in the heat transfer mechanism(s) accordingly, which are yet to be identified. For optimum overall performance of the microchannels, they must be designed for laminar flow conditions. But the laminar and transition heat transfer are unusual, and in these regimes *Nu* may be a function of other variables besides *Re*. In the laminar regime, *Nu* may recede with increasing *Re*, and in the transition regime *Nu* may be unaffected by *Re*, which are unexplained. The *Nu* in the laminar fully-developed and transition regimes also depends on fluid velocity and temperature, and geometry of the microchannel, but the nature of the dependence is unknown. Hence the survey indicates that something is amiss in the existing approach to convective heat transfer in microchannels. Through the following conventional dimensional analysis using inputs from the survey, it is found that the use of the Brinkman number can explain the unusual behaviour of heat transfer in the laminar and transition regimes.

2. DIMENSIONAL ANALYSIS BASED ON SURVEY

For laminar fully-developed flow through conventionally-sized channels, h depends on k and D_h. For microchannels, indications from the survey show that h depends on *Re*, and hence on ρ, V_{av}, and μ. In addition, h depends on the microchannel geometry, fluid properties, local wall and fluid temperatures. Hence δ, c, and ΔT should also be pertinent parameters to be considered. A general relationship between all the relevant quantities can be written as

$$f_1(h, k, \rho, V_{av}, \delta, D_h, \mu, c, \Delta T) = 0 \quad (1)$$

where f_1 is an unknown function. The dimensions of these quantities are as follows (here [ ] means 'dimensions of'):

$$[h] = M/\Theta T^3, \quad [k] = ML/\Theta T^3,$$

$$[\rho] = M/L^3, \quad [V_{av}] = L/T,$$

$$[\delta] = L, \quad [D_h] = L, \quad [\mu] = M/LT,$$

$$[c] = L^2/\Theta T^2, \quad [\Delta T] = \Theta$$

where M, L, T, and Θ are the basic dimensions of mass, length, time, and temperature, respectively.

As there are nine quantities and four basic dimensions, the Buckingham Π theorem [16] predicts the existence of five independent dimensionless groups, Π_1, Π_2, Π_3, Π_4, and Π_5. Hence, equation (1) may be written as

$$f_2(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5) = 0. \tag{2}$$

Adopting one approach in dimensional analysis [17], k, ρ, V_{av}, and D_h are chosen as the recurring parameters, and grouped to give the four fundamental dimensions as $M = [\rho D_h^3]$, $L = [D_h]$, $T = [D_h/V_{av}]$, $\Theta = [\rho V_{av}^3 D_h/k]$, the remaining five (principal) parameters may then be expressed in terms of the recurring parameters as follows:

$$[h] = M/\Theta T^3 = [k/D_h], \quad [\mu] = M/LT = [\rho V_{av} D_h],$$

$$[c] = L^2/\Theta T^2 = [k/\rho V_{av} D_h], \quad [\delta] = L = [D_h],$$

$$[\Delta T] = \Theta = [\rho V_{av}^3 D_h/k].$$

Therefore, the five dimensionless groups may be recast as:

$$\Pi_1 = h \cdot D_h/k, \quad \Pi_2 = \rho \cdot V_{av} \cdot D_h/\mu,$$

$$\Pi_3 = \rho \cdot V_{av} \cdot D_h \cdot c/k, \quad \Pi_4 = \delta/D_h,$$

$$\Pi_5 = k \cdot \Delta T/(\rho \cdot V_{av}^3 \cdot D_h). \tag{3}$$

Π_1 is Nu and Π_2 is Re. Π_3 and Π_2 may be re-grouped to form Pr $(=\Pi_3/\Pi_2)$.

The Π_5 and Π_2 may be re-grouped to form the Brinkman number $[Br = (\Pi_2 \cdot \Pi_5)^{-1} = \mu \cdot V_{av}^2/(k \cdot \Delta T)]$. It measures the relative importance of viscous heating to fluid conduction [18–20]. The Br also emerges from the dimensionless general energy equation (as the product of Ec and Pr) [21]. Although Br is usually neglected in low-speed and low-viscosity flows through conventionally-sized channels of short lengths, in flows through conventionally-sized long pipelines, Br may become important [20]. Since for flows through microchannels, the length-to-diameter ratios can be as large as for flows through conventionally-sized long pipelines, Br may become important in microchannels also. Hence equation (2) may be written as a correlation in the form of

$$Nu = A' \cdot (Re)^a \cdot (Pr)^b \cdot (\delta/D_h)^c \cdot (Br)^d. \tag{4}$$

It is noteworthy that the dimensionless number Π_4 has appeared implicitly in the literature. According to the proposal for laminar correlation by Peng and Peterson [13],

$$Nu = 0.1165 \cdot Re^{0.62} \cdot Pr^{1/3} \cdot (D_h/W_c)^{0.81} \cdot (H/W)^{-0.79}. \tag{5}$$

To bring H together with D_h, equation (5) can be approximated as

$$Nu = 0.1165 \cdot Re^{0.62} \cdot Pr^{1/3} \cdot (D_h/H)^{0.8} \cdot (W/W_c)^{0.8}. \tag{6}$$

In equation (6), the term $(D_h/H)^{0.8}$ is the same as $(\delta/D_h)^{-0.8}$ [i.e. $(\Pi_4)^{-0.8}$] since either H or W can be δ. The term (W/W_c) accounts for the geometry between two consecutive microchannels. Hence in general, equation (4) may be written as

$$Nu = A' \cdot (Re)^a \cdot (Pr)^b \cdot f_g(\delta, D_h) \cdot (Br)^d \tag{7}$$

where $f_g(\delta, D_h)$ is a function of the geometry of the individual microchannels and the microchannel structure (that is, the geometry between two consecutive microchannels), and a, b, d are the exponents of the dimensionless groups, and A' an empirical constant. For the laminar regime, using equations (6) and (7), a modified general form is

$$Nu = A' \cdot Re^{0.62} \cdot Pr^{1/3} \cdot f_g(\delta, D_h) \cdot (Br)^d. \tag{8}$$

3. COMPARISON OF VARIOUS DEFINITIONS OF THE BRINKMAN NUMBER

The Br has been used in certain viscous flow problems, especially those pertaining to lubrication. For instance, the Couette flow problem has been solved with the effect of Br [18, 19]. The ΔT in Br in these problems is the temperature difference between the two walls. In the Couette flow problem, the two walls are maintained at a constant temperature and hence the fully-developed temperature profile of the fluid remains unchanged (in form and value). Hence Br does not change along the flow.

For the constant wall temperature boundary condition, ΔT has been defined as the temperature difference between the local wall and the inlet fluid temperatures [20]. For this boundary condition, Br does not change along the fully-developed flow since the fluid temperature is unchanged.

For the constant wall heat flux boundary condition, which is applicable for the problem of cooling of IC chips, ΔT has been defined as the temperature difference between the local wall and fluid temperatures [20]. Thus even in the fully-developed region, Br will change due to the change in fluid viscosity, caused by a change in the fluid temperature.

4. ROLE OF THE BRINKMAN NUMBER IN MICROCHANNELS

4.1. *Physical significance*

For the case $\delta = H$ and a unit length in the fluid flow direction of a microchannel, Br may be re-written as

$$Br = \frac{\mu \cdot V_{av}^2}{k \cdot \Delta T} = \frac{\mu \cdot \left(\dfrac{2V_{av}}{\delta/2}\right) \cdot 2V_{av} \cdot 2W \cdot 1}{4k \cdot \left(\dfrac{\Delta T}{\delta/2}\right) \cdot 2W \cdot 1}. \tag{9}$$

The gradients are assumed to be linear since they are steep due to small δ. This approximation which is made for the profiles in lubrication problems [22], may be applicable for microchannels since the channel dimensions are of the order of or even smaller than the gaps for lubricants. For a linear velocity gradient, $V_{max} = 2V_{av}$ (where V_{max} is the fluid velocity at the axis of the microchannel). Hence the terms in the brackets in equation (9) are approximately the gradients, and equation (9) may be re-written as

$$4 \cdot Br = \frac{\left[\mu \cdot \left(\frac{\partial V}{\partial y}\right) \cdot V_{max} \cdot 2W \cdot 1\right]}{\left[k \cdot \left|\frac{\partial T}{\partial y}\right| \cdot 2W \cdot 1\right]} \quad (10)$$

where $(\partial V/\partial y)$ and $(\partial T/\partial y)$ are the fluid temperature and velocity gradients, respectively, along δ. The denominator in equation (10) is the heat transferred by conduction from the wall along δ and the numerator represents viscous dissipation due to a velocity gradient along δ, $[\int_0^{V_{max}} \mu \cdot (\partial V/\partial y) \cdot 2W \cdot 1 \cdot dV]$. Hence *Br* represents the ratio of the heat transferred from the wall by fluid conduction along the microchannel dimension to the work done against viscous shear, and is a measure of the effect of viscous dissipation relative to the heat transferred by conduction. From equation (10), *Br* is also seen to represent the ratio of the temperature and velocity gradients.

Since

$$\lim_{|\Delta T| \to 0} \frac{\Delta V}{|\Delta T|} = \left|\frac{\partial V}{\partial T}\right|$$

Br may also be written as

$$Br = \frac{\mu \cdot V_{av}}{2k} \cdot \left|\frac{\partial V}{\partial T}\right|. \quad (11)$$

Hence *Br* in microchannels is affected by the change of fluid velocity relative to the temperature along the microchannel dimension.

The role of *Br* in microchannels is better appreciated by considering the case of the fluid cooled, where a simple energy balance gives (neglecting viscous dissipation due to velocity gradient along the other direction),

$$\dot{m}_f \cdot c \cdot \Delta T_f = h \cdot 2W \cdot 1 \cdot (T_f - T_w) =$$

$$h' \cdot 2W \cdot 1 \cdot (T_f - T_w) - \mu \cdot \frac{\partial V}{\partial y} \cdot 2W \cdot 1 \cdot V_{max}. \quad (12)$$

In equation (12), the net heat transfer coefficient h includes the relative effect of viscous dissipation, whereas the convective heat transfer coefficient h' does not include the effect due to *Br* since the viscous heat generation term is considered separately. Since $h' \cdot (T_f - T_w) = k \cdot |\partial T/\partial y|$, equation (12) may be re-arranged to give

$$h = \frac{\dot{m}_f \cdot c \cdot \Delta T_f}{(2W \cdot 1) \cdot (T_f - T_w)} = \frac{2k}{\delta} \cdot \left(1 - 4 \cdot \left[\frac{\mu \cdot V_{av}^2}{k \cdot (T_f - T_w)}\right]\right) \quad (13)$$

where the term within the square brackets is *Br*. Since $D_h = 2 \cdot \delta$ for the 1-D case under consideration, equation (13) may also be written as

$$Nu = 4 \cdot (1 - 4 \cdot Br). \quad (14)$$

As $Br \to 0.25$, the effect of cooling the fluid is offset by viscous dissipation and the wall heat transfer has no effect on the fluid temperature. *Br* is likely to increase as δ reduces, since ΔT will reduce due to the lower conduction resistance of the fluid. Hence the temperature gradients may not change much or may even reduce as $\delta \to 0$. However, a given small D_h would enable laminar flow at larger velocities, which leads to a higher *Re*, thereby enhancing the wall heat transfer as per equation (8). Hence the fluid velocity may be increased causing $(\partial V/\partial T)$ to increase. Therefore from equation (11), *Br* will increase and as $Br \to 0.25$, at which point the importance of viscous dissipation would be comparable to the wall heat transfer. Hence if the effect due to *Br* is not considered, the gains due to optimisation of the microchannel heat sink configuration which has been presented for instance by Tuckermann and Pease [4], may not be realised. The increase in viscous dissipation due to reduction in δ is analogous to the increase in the heat generation associated with the reduction in the feature size of the circuits in an IC chip since it causes a higher resistance to the current flow, whereas in microchannels the increase in fluid temperature due to viscous dissipation is due to the increase in the fluid flow resistance accompanied by a reduction in δ. It follows that the problem of heat dissipation accompanied by reducing the feature size of IC chips cannot be solved by indefinitely reducing the microchannel dimensions without considering the effect due to *Br*, as this will lead to the same problem of viscous heat dissipation in the fluid which will offset the gains of high h associated with a reduction in δ. In short, viscous dissipation decides the fundamental limit to the reduction of the microchannel dimension, and *Br* is a measure of that limit.

Though an energy balance across the microchannel indicates a correlation between *Nu* and *Br*, equation (14) may not be applicable for very small *Br* since *Nu* is predicted by it to be a constant. Nevertheless, since *Br* does appear in the energy balance and also emerges from a dimensional analysis along with other numbers, it is worth exploring the possibilities of the various forms of correlations with *Br*.

4.2. *Role in heat transfer*

The *Br* is significant in the laminar regime since the steep gradients are maintained in microchannels, and the relative effect of viscous dissipation may become important. Its importance in the transition regime may be less because the steep gradients may not be maintained throughout the cross-section. In the turbulent regime, the gradients are irregular and other mechanisms like the bursting of turbulent eddies (as discussed by Yu *et al.* [10]) may dominate the heat transfer, and hence the importance of *Br* is further reduced. *Br* has no relevance in the conventionally-sized channels, since for the same gradients as in the microchannels, the velocities required would be much larger, and prior to attaining these large velocities, the flow mode changes to turbulent, thereby flattening the gradients.

Viscous dissipation, being a source term in the fluid flow, reduces the cooling capacity of the fluid since the temperature rise of the fluid is also contributed by the conversion of the pumping power to heat, and not due to the heat transferred from the wall alone. Therefore the increase in h due to the effect of viscous dissipation is misleading, since the cooling capacity may have actually been reduced.

Since *Br* features the relative importance of viscous dissipation, it plays opposite roles for the cases of fluid heated and cooled. For the cooling of IC chips, the fluid is heated and the exponent d of *Br* in equation (8) should be positive, since the $(\mu \cdot V^2)$ term shall also tend to increase the coolant temperature and hence *Nu*. At a constant velocity, *Re* increases due to reduction in the coolant viscosity because of the temperature rise of the fluid. Hence the $(\mu \cdot V^2)$ term decreases and $Re^{0.62} \cdot Br^d$ (where d is positive) decreases. This explains the reduction of *Nu* though *Re* increases in the laminar regime as observed [9, 11]. But this reduction of *Nu* being due to the reduced importance of viscous dissipation relative to the wall heat transfer, increases the capacity of cooling the chip surface. However if *Re* increases because of an increase in velocity at constant viscosity (constant fluid temperature), the importance of viscous dissipation relative to the wall heat transfer will increase. Thus $(\mu \cdot V^2)$ and $Re^{0.62} \cdot Br^d$ will increase, and *Nu* will increase with *Re* in the laminar regime.

For the case of the fluid cooled, the exponent d of *Br* in equation (8) should be negative, since the $(\mu \cdot V^2)$ term increases the coolant temperature and hence reduces *Nu*. At a constant velocity, *Re* decreases due to increase in the coolant viscosity because of the fluid being cooled. Hence the $(\mu \cdot V^2)$ term increases causing *Br* to increase thereby increasing the relative importance of viscous dissipation, and it follows that $Re^{0.62}/Br^d$ (where $-d$ is positive) also reduces. In this case *Nu* decreases with decreasing *Re*, but the heating capacity increases. However, if *Re* reduces because of a decrease in velocity at a constant viscosity (constant fluid temperature), the relative importance of viscous dissipation will decrease. Therefore *Br* will reduce and $Re^{0.62}/Br^{-d}$ may increase, and consequently *Nu* may increase though *Re* reduces. But the heating capacity will decrease.

Thus, the conclusion for both the cases of fluid heated and cooled, the variation of the viscosity with temperature is beneficial to the heat transfer since the cooling and heating capacities, respectively, increase. The role of *Br* in the laminar regime heat transfer as presented in the above arguments is summarised by the flow chart shown in Fig. 1.

In the transition regime, viscous dissipation and therefore *Br* are relatively less important since the gradients are not maintained throughout the cross-section. Hence the absolute value of the exponent d of *Br* will be lower than that for the laminar regime, and may explain why $Re^{0.62} \cdot Br^d$ and hence *Nu*, was found to be approximately constant with *Re*, for the case of the fluid heated [9]. For the case of the fluid cooled, the product $Re^{0.62} \cdot Br^d$ and therefore *Nu* will reduce but at a lower rate compared to the laminar regime.

5. CORRELATING WITH EXPERIMENTAL DATA

The laminar regime experimental data points available from the survey which can be used for testing the

Table 1. Experimental data in the laminar regime

T_w (°C)	Re (–)	Nu (–)	$\nu \times 10^7$ (m² s⁻¹)	T_f (°C)	k (W m⁻¹ K⁻¹)	Pr (–)	ΔT (K)	Br $\times 10^5$	$Nu/(Re^{0.62} \cdot Pr^{1/3})$ $\times 100$
Case 1 : $V_{av} = 0.25$ m s^{-1}, $T_{f_{in}} = 19.7$°C									
25.0	80	0.46	9.736	21.5	0.605	6.71	3.5	2.8333	1.599
30.5	87	0.40	8.918	25.3	0.612	6.07	5.0	1.8109	1.366
41.0	97	0.39	7.973	30.6	0.620	5.34	10.6	0.7532	1.297
51.0	107	0.35	7.243	35.2	0.627	4.80	15.7	0.4559	1.145
Case 2 : $V_{av} = 0.29$ m s^{-1}, $T_{f_{in}} = 18.8$°C									
25.0	93	0.49	9.625	22.0	0.606	6.62	3.0	4.4115	1.570
28.5	100	0.46	8.990	25.0	0.611	6.13	3.4	3.5956	1.456
36.0	116	0.41	7.743	31.9	0.622	5.16	4.3	2.4100	1.236
41.5	124	0.37	7.268	35.1	0.627	4.82	6.7	1.4541	1.107
47.0	136	0.36	6.635	39.8	0.633	4.35	7.2	1.2091	1.057
Case 3 : $V_{av} = 0.76$ m s^{-1}, $T_{f_{in}} = 19.3$°C									
25.1	250	0.82	9.462	22.8	0.607	6.50	2.3	39.100	1.425
30.6	273	0.70	8.658	26.6	0.614	5.87	4.0	20.133	1.189
36.0	300	0.60	7.881	31.1	0.621	5.27	4.9	14.853	1.003

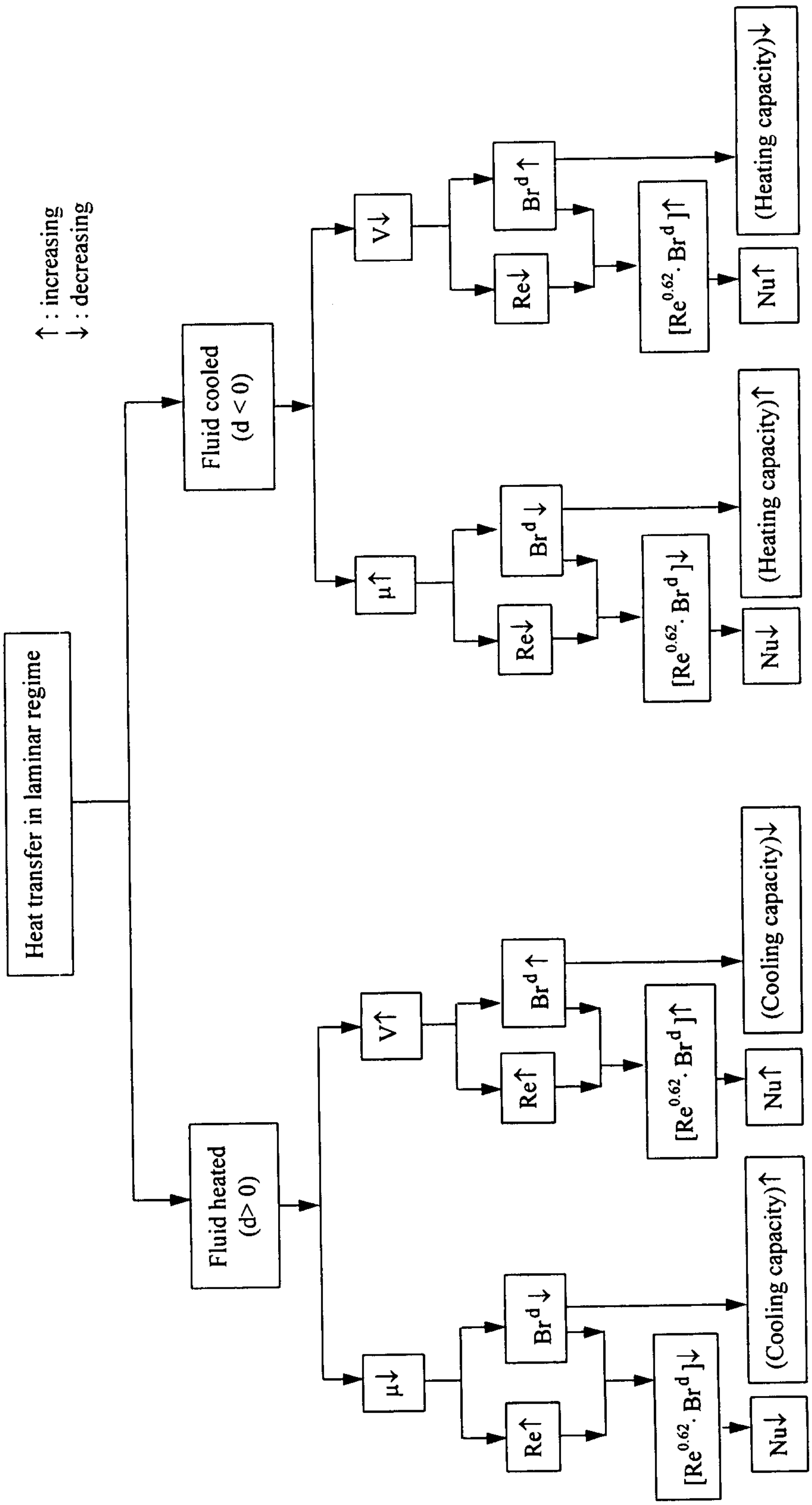

Fig. 1. Flow chart illustrating the role of *Br* in explaining the unusual heat transfer behaviour in the laminar regime.

validity of Br are rather limited. Wang and Peng [9] have presented the experimentally obtained variations of h with T_w and of Nu with Re for different microchannel geometries, fluid inlet temperatures, and velocities, for water flow. Only their reported data can be processed to give Br and hence the correlations, since a one-to-one correspondence exists between Nu, Re, and T_w, in the data presented. However, most of their data belong to the transition regime, and laminar data necessary for the correlation with Br is available for only three different fluid velocities for a fixed microchannel geometry, giving a total of 12 data points. Their data are extracted from Figs. 3(f) and 6(d) from reference [9], and shown in Table 1 as Cases 1, 2, and 3, respectively. However Case 3 has only three laminar data points. In Table 1, the data in the first three columns (T_w, Re, and Nu) are as reported in [9]. The v is found from Re, and T_f is found by interpolation from v using the data for standard properties of water obtained from [23]. Then k and Pr are also obtained by interpolation from T_f using the standard properties. The Br then calculated from the experimental data is also tabulated, and is found to be much smaller than the fundamental limit of 0.25 discussed earlier.

Since for these cases, the microchannel geometry is fixed ($D_h = 0.31$ mm), the geometric function $f_g(\delta, D_h)$ in equation (8) can be absorbed in the empirical constant A, yielding the form

$$Nu = A \cdot Re^{0.62} \cdot Pr^{1/3} \cdot Br^{d}. \quad (15)$$

5.1. *Comparison of data with existing and proposed correlations*

If the correlation is of the form of equation (15), the plot of $[Nu/(Re^{0.62} \cdot Pr^{1/3})]$ vs. Br should show a linear form on a log scale. The plots for the three cases are shown in Fig. 2(a), along with the best fit for all the points combined. The experimental data points for the individual cases approximately follow a linear relation and the best fits are positively sloped, justifying the positive exponent of Br for the case of the fluid heated being considered. From the graphs, the data for the separate cases appear to have a better linear trend than the combined case, although the lack of data preclude further speculations.

In order to compare with the existing correlation form [equation (5)], the plot of $Nu/Pr^{1/3}$ vs. Re for the same data is shown in Fig. 2(b), and of Nu vs. Re in Fig. 3. Although in general $(Nu/Pr^{1/3})$ reduces as Re increases, two points, each for Cases 1 and 2, appear to be out of this trend. However, the same two points do not appear to misbehave in the plot of Fig. 2(a). More interestingly, it is noted that the gradients of the individual lines in Figs. 2(b) and 3 are not near to +0.62, as predicted by equation (5), hinting again that the form of equation (5) applied to individual cases may be incomplete.

In order to explore the use of the form of equation (14), Nu vs. Br is plotted in Figs. 4(a) and (b) on

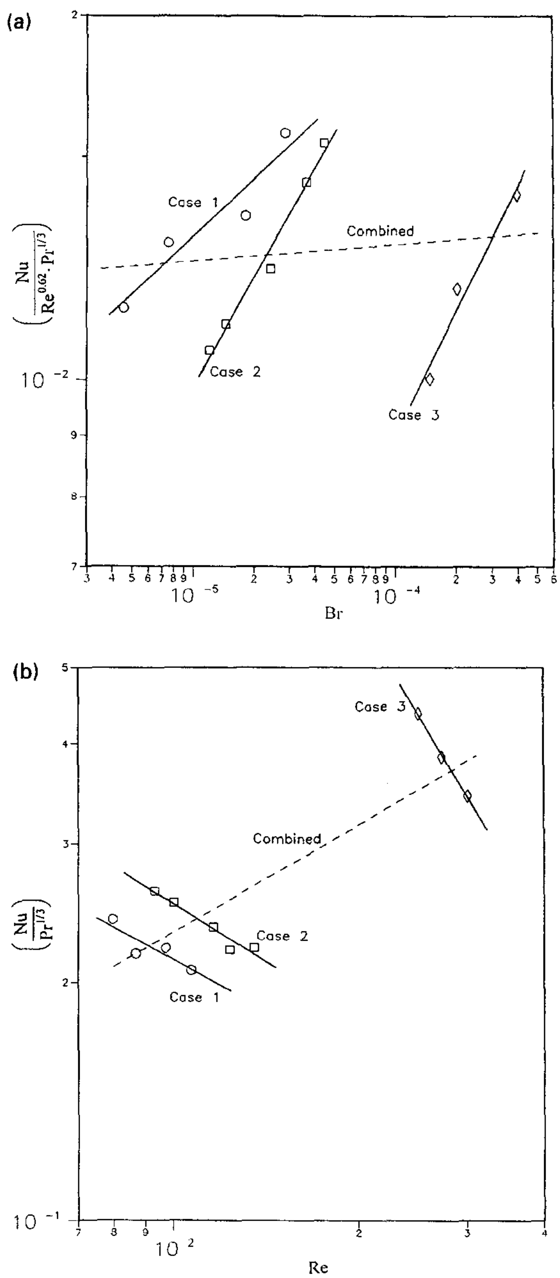

Fig. 2. (a) Plot of $[Nu/(Re^{0.62} \cdot Pr^{1/3})]$ vs. Br. (b) Plot of $Nu/Pr^{1/3}$ vs. Re.

log and linear scales, respectively. Again for both the figures the plots have a positive slope.

5.2. *Quantitative comparison of the correlations*

In order to quantify the obedience of the linear form, the values of the slopes and intercepts of the correlations together with their uncertainties for all the plots are given in Table 2, where the uncertainties σ are the square root of the variance [24]. The uncertainties in slopes and intercepts are generally higher for the correlations without Br for individual and

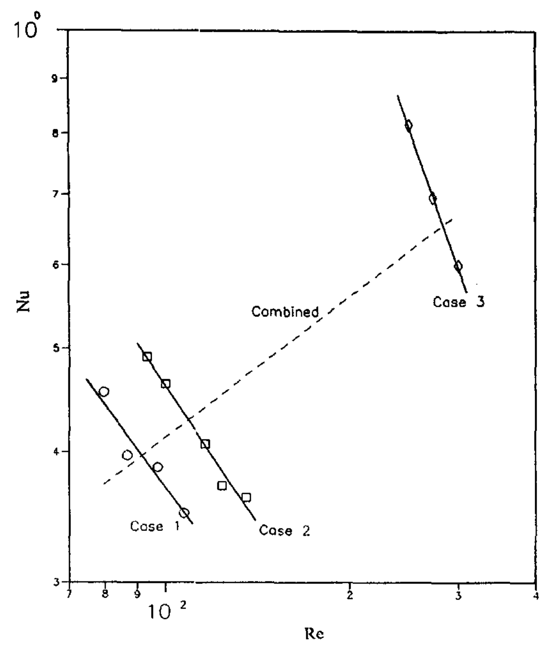

Fig. 3. Plot of *Nu* vs. *Re*.

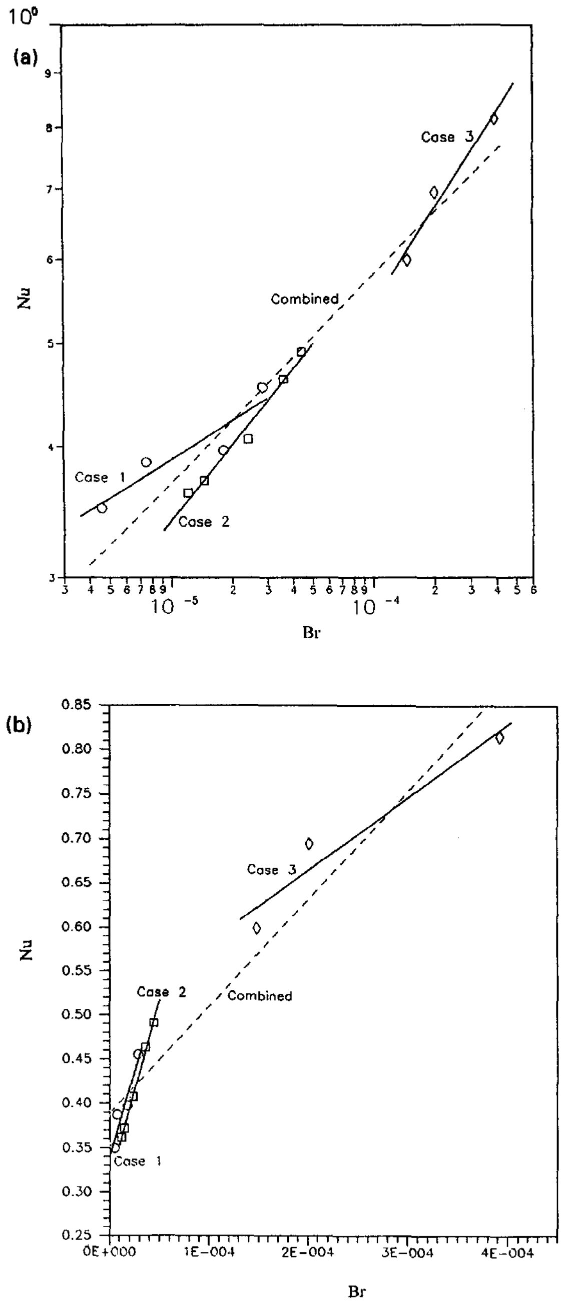

Fig. 4. (a) Plot of *Nu* vs. *Br*. (b) Plot of *Nu* vs. *Br* on a linear scale.

combined cases. By comparing the correlations for Fig. 4(a) and (b) it is seen that the individual cases may fit better on a linear scale but the combined cases may fit better on a log scale. This hints at a piecewise linear fit for small *Br* ranges and a power fit for a wider *Br* range, suggesting varying significance of *Br* for different values of *Br*. The significance of *Br* is expected to diminish for low values of *Br*.

Though the data available is insufficient to arrive at a universal correlation between *Nu* and *Br*, the comparison of the correlation of the available data with and without *Br* indicate the existence of a correlation between *Nu* and *Br* in the laminar regime, and suggests that the existing correlation form may be inadequate. To arrive at a universal correlation, comprehensive experimental data are needed in the laminar regime spanning a wider range of *Br*. The exponent *d* in equation (14) may also vary for different ranges of *Br*; at least the absolute value of *d* is expected to decrease as *Br* reduces, since the importance of *Br* is expected to diminish then. With respect to the geometric function $f_g(\delta, D_h)$, perhaps a similar form as that proposed in equation (6) may be adopted. Thus, using the dimensionless geometric parameter $(D_h \cdot W/H \cdot W_c)$, equation (7) may be put as

$$Nu = A' \cdot (Re)^a \cdot (Pr)^b \cdot \left(\frac{D_h \cdot W}{H \cdot W_c}\right)^c \cdot (Br)^d \quad (16)$$

where the exponent *c* needs to be determined experimentally by using microchannels of different geometry. The corresponding form of the geometric parameter for circular microchannels of diameter *D* would be (D/W_c). Finally, the role of *Br* may also be important in transition from laminar to turbulent flow, and this is presently under investigation.

6. CONCLUSIONS

1. Elementary dimensional analysis using inputs from the survey shows that in addition to *Re*, *Pr*, and a dimensionless geometric parameter of the microchannels, *Nu* may correlate with *Br* in the form of

Table 2. Slopes and intercepts of the correlations and their uncertainties in Figs. 2(a), 2(b), 3, 4(a), 4(b)

Experimental case	Figure number	Slope	σ_{slope}	Intercept	$\sigma_{intercept}$
1	2(a) : $\left[\frac{Nu}{Re^{0.62} \cdot Pr^{1/3}} \text{ vs. } Br\right]$	0.16	± 0.03	-1.1	± 0.1
	2(b) : $\left[\frac{Nu}{Pr^{1/3}} \text{ vs. } Re\right]$	-0.4	± 0.1	0.2	± 0.2
	3 : [*Nu* vs. *Re*]	-0.8	± 0.1	1.2	± 0.2
	4(a) : [*Nu* vs. *Br*]	0.12	± 0.02	0.2	± 0.1
	4(b) : [*Nu* vs. *Br*], linear scale	3800	± 600	0.34	± 0.01
2	2(a) : $\left[\frac{Nu}{Re^{0.62} \cdot Pr^{1/3}} \text{ vs. } Br\right]$	0.30	± 0.02	-0.5	± 0.09
	2(b) : $\left[\frac{Nu}{Pr^{1/3}} \text{ vs. } Re\right]$	-0.49	± 0.06	0.4	± 0.1
	3 : [*Nu* vs. *Re*]	-0.87	± 0.06	1.4	± 0.1
	4(a) : [*Nu* vs. *Br*]	0.24	± 0.01	0.72	± 0.07
	4(b) : [*Nu* vs. *Br*], linear scale	4100	± 90	0.312	± 0.003
3	2(a) : $\left[\frac{Nu}{Re^{0.62} \cdot Pr^{1/3}} \text{ vs. } Br\right]$	0.35	± 0.04	-0.7	± 0.1
	2(b) : $\left[\frac{Nu}{Pr^{1/3}} \text{ vs. } Re\right]$	-1.30	± 0.04	2.77	± 0.09
	3 : [*Nu* vs. *Re*]	-1.69	± 0.04	3.95	± 0.09
	4(a) : [*Nu* vs. *Br*]	0.30	± 0.03	1.0	± 0.1
	4(b) : [*Nu* vs. *Br*], linear scale	800	± 100	0.50	± 0.03
1, 2, 3, combined	2(a) : $\left[\frac{Nu}{Re^{0.62} \cdot Pr^{1/3}} \text{ vs. } Br\right]$	0.01	± 0.03	-1.8	± 0.1
	2(b) : $\left[\frac{Nu}{Pr^{1/3}} \text{ vs. } Re\right]$	0.45	± 0.08	-1.5	± 0.2
	3 : [*Nu* vs. *Re*]	0.4	± 0.1	-1.3	± 0.2
	4(a) : [*Nu* vs. *Br*]	0.20	± 0.01	0.55	± 0.06
	4(b) : [*Nu* vs. *Br*], linear scale	1200	± 100	0.39	± 0.13

equation (4) generally, and in the form of equation (16) in particular, for rectangular microchannels.

2. The *Br* is not only a measure of the relative importance of viscous dissipation in microchannels, but also decides the fundamental limit for the reduction of the microchannel dimension. It should be more important in the laminar regime compared to the transition and turbulent regimes.
3. For the case of fluid heated the exponent of *Br* is positive and it has been confirmed by experimental data. From the present analysis, the exponent of *Br* should be negative for fluid cooled.
4. The unusual behaviour of *Nu* receding with *Re* increasing in the laminar regime can be explained by *Br*.
5. Comparison with limited available experimental data in the laminar regime indicates support for *Br*. However comprehensive data are required to obtain a universal correlation.

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